# Quantum Ground State Estimation of SU(2) Gauge Theory using Variational Quantum Eigensolver

A. Suneesh, K. Liegener[1, 2], M. Knudsen[1], M. Singh[1], M. Werninghaus[1], F. Roy[1,3] and S. Filipp[1, 2]

[1]Walther-Meißner-Institut, Bayerische Akademie der Wissenschaften, 85748 Garching, Germany
[2]School of Natural Sciences, Technische Universität München, 85748 Garching, Germany
[3]Theoretical Physics, Saarland University, Saarbrücken 66123, Germany

## Motivation

Ground state problem for interacting Lattice Gauge Theories (LGT) is challenging due to **entanglement** over spaced lattice sites. Numerical simulations break down at certain scales due to classical computers being **incapable** of simulating large Hilbert spaces.
⇒ **Can quantum computers help to fill gaps in fundamental science**?

Variational Quantum Algorithms (VQE) struggle to show quantum advantage due to hybrid methods often requiring many readout operations, which scale unfavorably with system size, and having to sample huge parameter spaces.
⇒ **Can ground state symmetries in LGT resolve these challenges**?

**Goal** : to use a VQE to find the ground state energy of a simplified SU(2) Yang-Mills theory

## SU(2) Yang-Mills theory

- Weak force is described as a SU(2) gauge theory with Yang-Mills Hamiltonian.
- Bare discretization for LGT via the Kogut-Susskind-Hamiltonian with coupling constant $g(\epsilon) < 1$ reads

$$H_{Standard-model} = H_{SU(3)} + \boxed{H_{SU(2)}} + H_{U(1)} + H_{matter}$$

$$H_{SU(2)} = \int_{\mathbb{R}^3} d^3x \left(\frac{\kappa}{2} E_I^a E_a^I + \frac{1}{4\kappa} F_{ab}^I F_I^{ab} - \lambda\, G_x\right)$$

$$\approx \sum_{\vec{v}\in\mathbb{Z}^3} \frac{\kappa}{2\epsilon}\left(\sum_{k=1,..,3} P_I(v,k)P^I(v,k) - 2g(\epsilon)\sum_{\square} h(\square_v) - \lambda\, G_v\right)$$

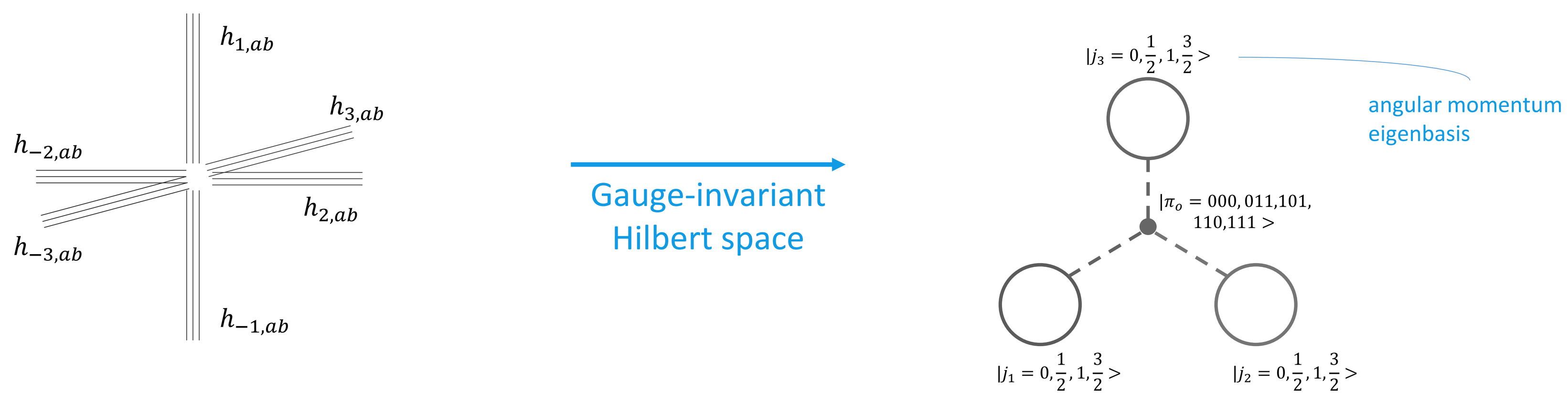
with the Gauss constraint $G_x = \partial_a E_I^a + \epsilon_{IJK} A_a^J E_K^a$ and the Wilson-loops $h$(e) as gauge-invariant functions.

- To make the problem suitable for near-term quantum devices, we focus on the simplest case: a single lattice vertex with periodic boundary conditions.
- The Hamiltonian is translated into a form usable by a quantum computer by decomposing it into electric ($H_{el}$) and magnetic ($H_{mag}$) components, which are expressed as a sum of Pauli string operators (combinations of X, Y, Z, and I gates).
- The specific Hamiltonian whose ground state we seek is a function of a coupling parameter x:

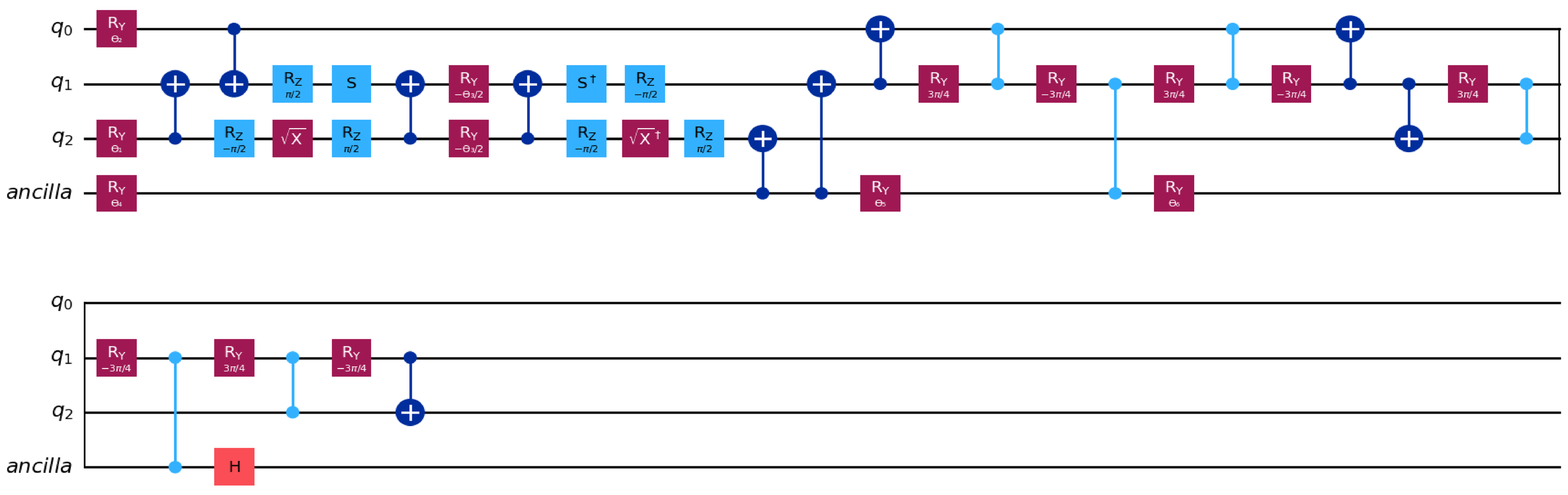
$$H(x) = (1 - x^2)H_{\{el\}} - 2x^2 H_{\{mag\}}$$

## State Preparation with Quantum Algorithms

- **Decomposition and Truncation**: This physical space is not uniform; it further decomposes into independent superselection sectors, meaning we can solve for the eigenstates in each sector separately. The ground state of the Hamiltonian resides in a specific sector known as the **θ-graph**, where the physical states are uniquely defined by **three angular momentum quantum numbers** ($j_1$,$j_2$,$j_3$). For practical simulation on near-term hardware, we then truncate this space to a maximum angular momentum of $\mathbf{j_{max}=\frac{1}{2}}$.
- **Isolating the Ground State Sector:** The truncation to a low energy cutoff, $j_{max}=\frac{1}{2}$, was chosen because the system's energy levels converge rapidly as the cutoff increases, meaning we can accurately model the low-energy properties without simulating the full, infinite-dimensional Hilbert space. By isolating this sector, we concentrate our computational efforts on the most relevant part of the physical problem.



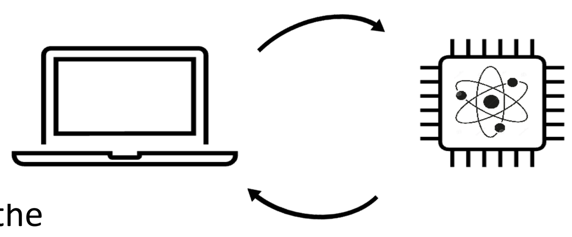

- **Mapping the Physical System to Qubits**: This truncated, physically constrained system is mapped onto a 4-qubit architecture for the VQE algorithm:
  - 3 Physical Qubits encode the state of the three quantum numbers ($j_1$, $j_2$, $j_3$)
  - Ancilla Qubit controls circuit dynamics.
    - Mediates interactions such as excitations and swaps between the physical qubits.
    - Enforces symmetry constraints of the system.
    - Final measurements are post-selected on the ancilla being in the |0⟩ state, projecting the final quantum state back onto the valid physical subspace.




VQE ansatz, a problem-specific circuit designed to prepare trial ground states for the , $j_{max}$=½ system using 6 variational parameters

## VQE algorithm – Shots-based Analysis

- Variational Quantum Eigensolver (VQE) is a hybrid algorithm, where the parameters of a quantum algorithm are varied by a classical optimizer.
  - A quantum processor prepares a trial quantum state, | Ψ(θ) ⟩, based on a set of parameters, θ.
  - A classical optimizer then adjusts these parameters to minimize the expectation value of the Hamiltonian, ⟨Ψ(θ) | H | Ψ(θ)⟩



- Finds ground state using $E_0 \leq \langle\psi|H|\psi\rangle$
  - Create parameterized $|\psi(\theta)\rangle$ and minimize $\langle\psi(\theta)|H|\psi(\theta)\rangle$
- Enables work with short circuits and could find potential application in NISQ
- Parameterized quantum circuit, or "ansatz," efficiently represents the ground state.
- Our simulation uses a problem-specific circuit designed for the , $j_{max}$=½ SU(2) system.
- This 4-qubit circuit (3 physical + 1 ancilla) contains 6 tunable parameters ($\theta_1$, …, $\theta_6$) and is built from a sequence of excitation and swap gates designed to explore the valid physical state space.

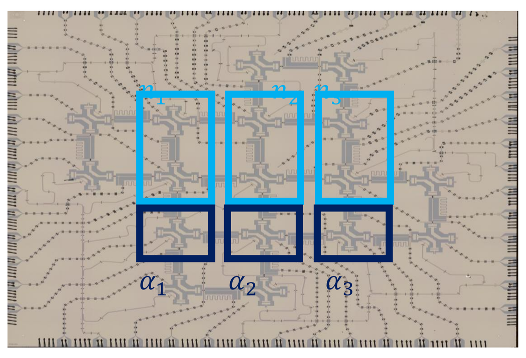
**Simulating Realistic Measurements: Shot-Based Analysis**
While ideal simulators compute exact energy, real quantum computers rely on statistical sampling. We simulate this process to predict performance on actual hardware.

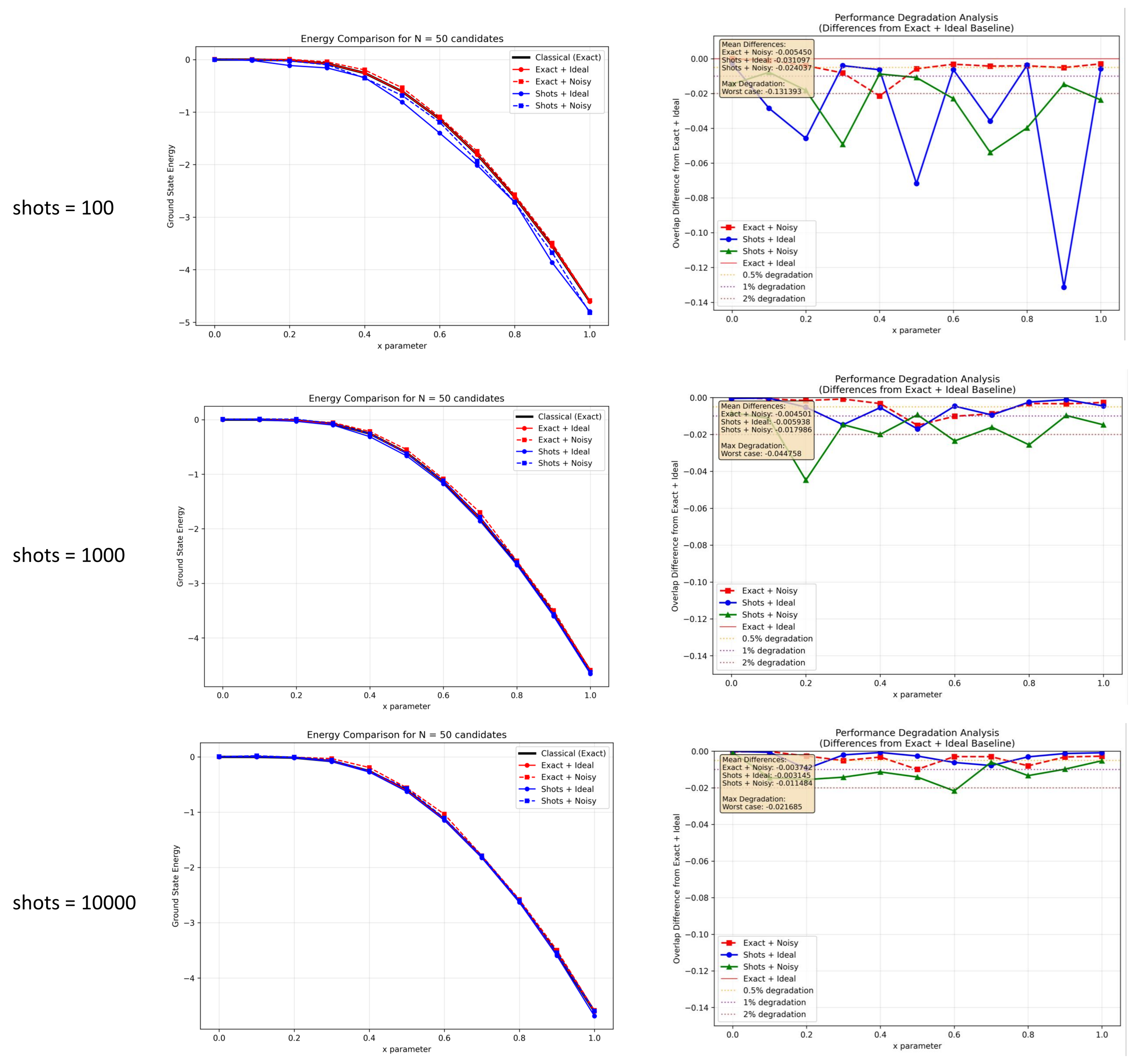
The Hamiltonian is a weighted sum of many Pauli strings: $H(x) = \Sigma_i c_i(x) P_i$.
We cannot measure the total energy at once. Instead, for each Pauli string $P_i$ (e.g., XZI):
- Add appropriate basis-change gates (like H or S†) at the end of the circuit.
- Execute the circuit for a fixed number of shots (e.g., 10,000 times).
- Compute ⟨$P_i$⟩ from the resulting measurement counts of 0s and 1s.
- Reconstruct the final energy classically: $\mathbf{E = \Sigma_i c_i(x) \langle P_i\rangle}$.

**Enforcing Symmetry via Post-Selection**: For every shot, we only keep the measurement outcomes where the ancilla qubit is |0⟩. This post-selection projects the result back into the valid, gauge-invariant physical Hilbert space.
**Impact on Optimization**: Shot-based approach introduces sampling noise, making the energy landscape "bumpy" for the optimizer. Higher number of shots reduces this noise but increases simulation time.

## Implementation on NISQ devices

- Technical requirements:
  - At least 4 qubits – 3 physical qubits + 1 ancilla bit
  - Native 2-Qubit-Gate Set: CZ
  - Assumed nearly perfect readout
  - 1-Qubit gate fidelities 99.9%
  - 2-Qubit gate fidelities 99.5%
  - Idling fidelities: 99.9% per qubit in parallel to a 2-qubit gate




- The energy landscape for the VQE optimization becomes smoother as the number of measurement shots increases, reducing sampling noise.
- Prepared state is available for further physical experiments
- With 7 qubits one could also investigate the full 6-valent vertex Hilbert space with similar techniques

## Outlook

- Experimental implementation of this VQE simulation on a superconducting quantum device at the Walther-Meißner-Institut (WMI). The simulation framework is prepared for deployment, with execution planned on the 17-qubit chip.

## Acknowledgments

This project was completed as part of the MCQST Summer Bachelor Program.